Mathematricks

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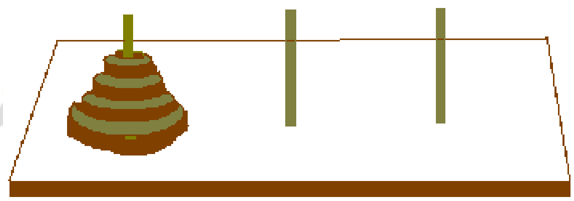
In the previous issue we concluded with Towers of Hanoi introduction.

Towers of Hanoi problem: It is a mathematical game or puzzle. It consists of three rods, and a number of disks of different sizes which can slide onto any rod. Initially the disks stacked in order of size on one rod, the smallest one at the top. The objective of the puzzle is to move the entire stack to another rod, obeying the following rules.



- ***Only one disk may be moved at a time.***
- ***Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod.***
- ***No disk may be placed on top of a small disk.***

Our goal is to move the disks from one pole to another with the help of the intermediate (temporary) pole.

Let us start with one disk initially. It requires only one move from source to destination. Intermediate pole is not required. Let us use S for Source and D for destination and I for intermediate poles. Subscript represents disk number from top to bottom. So I_2 can't be on I_1. D_2 can't be on D_1. The moves for two disks, three disks and four disks are shown below. Based on those moves we will write an algorithm.

One disk	Two disks	Three disks
$S_1 \rightarrow D_1$	$S_1 \rightarrow I_1$ $S_2 \rightarrow D_2$ $I_1 \rightarrow D_1$	$S_1 \rightarrow D_1$ $S_2 \rightarrow I_2$ $D_1 \rightarrow I_1$ $S_3 \rightarrow D_3$ $I_1 \rightarrow S_1$ $I_2 \rightarrow D_2$ $S_1 \rightarrow D_1$

Four disks:

$S_1 \rightarrow I_1$; $S_2 \rightarrow D_2$; $I_1 \rightarrow D_1$; $S_3 \rightarrow I_3$; $D_1 \rightarrow S_1$; $D_2 \rightarrow I_2$;
$S_1 \rightarrow I_1$; $S_4 \rightarrow D_4$; $I_1 \rightarrow D_1$; $I_2 \rightarrow S_2$; $D_1 \rightarrow S_1$; $I_3 \rightarrow D_3$;
$S_1 \rightarrow I_1$; $S_2 \rightarrow D_2$; $I_1 \rightarrow D_1$;

If we notice carefully, One disk corresponds to one move $(2^1 - 1)$; Two disks corresponds to 3 moves $(2^2 - 1)$; Three disks corresponds to 7 moves $(2^3 - 1)$; Four disks corresponds to 15 moves $(2^4 - 1)$; In this way 'n' disks corresponds to $(2^n - 1)$ moves.

Other observations:

If the number of disks are odd, the first disk is moved to Destination. If the number of disks are even, the first disk is moved to Intermediate pole.

One disk corresponds to One move. (3 x 0 + 1)
Two disks corresponds to Three moves. (3 x 1)
Three disks corresponds to Seven moves. (6 x 1 + 1)
Four disks corresponds to Fifteen moves. (3 x 5)
Five disks corresponds to Thirty One moves. (6 x 5 + 1)
Six disks corresponds to Sixty Three moves. (3 x 21)
Seven disks corresponds to One hundred Twenty Seven. (6 x 21 + 1)

Always the large disk is moved to the destination exactly halfway.

Let us assume there are 3 disks (top one being 1 and bottom most being 3) to move from Peg A to Peg C using the temporary peg B. Now to solve this, assume disks 1 and 2 as a single disk. Then that goes to temporary peg B and the disk 3 goes to peg C. But getting disks 1 and 2 onto peg B is not a single step. First disk 1 is to be moved to peg C, disk 2 is moved to peg B and disk 1 is moved to peg B. It is a recursive process.

We will see about the recursive process and how this problem can be solved using recursive process in the next edition.

(Let us continue in the next issue, till then good bye!)